SM223 – Calculus III with Optimization Fall 2017
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Review Quiz 1

Instructions. You have 20 minutes to complete this review quiz. You may use your calculator. You may not use any other materials. Put your answers on the separate answer form provided.

1. If the cross product of two nonzero vectors is $\langle 0,0,0\rangle$, what can we conclude about the vectors?
   (a) Nothing – not enough information.
   (b) They are orthogonal.
   (c) They are parallel.
   (d) They are unit vectors.
   (e) The vectors have the same magnitude.

   Recall: $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$.
   If $\vec{a}\times\vec{b} = \langle 0,0,0\rangle$ and $\vec{a}\neq\vec{0}$, $\vec{b}\neq\vec{0}$, then $\sin\theta = 0 \Rightarrow \theta = 0 \Rightarrow \vec{a}$ and $\vec{b}$ are parallel

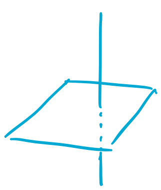
2. Which of the following is a unit vector?
   (a) $\langle 2,1,-2\rangle$
   (b) $\langle 2/3,1/3,-2/3\rangle$
   (c) $\langle 2/5,1/5,2/5\rangle$
   (d) $\langle 1,1,1\rangle$
   (e) $\langle 1/3,1/3,1/3\rangle$

   $$\left|\left\langle \tfrac{2}{3}, \tfrac{1}{3}, -\tfrac{2}{3}\right\rangle\right| = \sqrt{\tfrac{4}{9}+\tfrac{1}{9}+\tfrac{4}{9}} = 1$$

3. Which vector is orthogonal to $\langle 1,3,2\rangle$?
   (a) $\langle 1,1,1\rangle$
   (b) $\langle 0,1,0\rangle$
   (c) $\langle 1,-1,1\rangle$
   (d) $\langle -1,0,1\rangle$
   (e) $\langle 2,3,1\rangle$

   Recall: $\vec{a}$ and $\vec{b}$ are orthogonal if $\vec{a}\cdot\vec{b}=0$
   $$\langle 1,3,2\rangle\cdot\langle 1,-1,1\rangle = 1-3+2 = 0$$

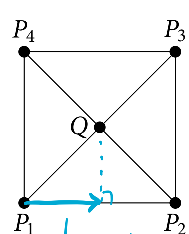
4. Which of these planes is perpendicular to the line $x = 2 - t,\ y = -2 + \frac{1}{2}t,\ z = 1 + 2t$?

   (a) $x - \frac{1}{2}y - 2z = 5$
   (b) $2x - 2y + z = 3$
   (c) $x - 2y - \frac{1}{2}z = 8$
   (d) $-\frac{1}{2}x + \frac{1}{2}y - z = 7$
   (e) $2x + z = 4$

   This line has direction vector $\langle -1, \frac{1}{2}, 2\rangle$
   This plane has a normal vector $\langle 1, -\frac{1}{2}, -2\rangle$ → These 2 vectors are parallel

5. For this configuration of points, what is the vector projection of $\overrightarrow{P_1Q}$ onto $\overrightarrow{P_1P_2}$?



$\text{proj}_{\overrightarrow{P_1P_2}}\, \overrightarrow{P_1Q}$

   (a) $\overrightarrow{P_1P_2}$
   (b) $2\overrightarrow{P_1P_2}$
   (c) $\frac{1}{2}\overrightarrow{P_1P_2}$
   (d) $\sqrt{2}\,\overrightarrow{P_1P_2}$
   (e) $\frac{1}{\sqrt{2}}\overrightarrow{P_1P_2}$